Part I

Module - 1 - Oscillations and Shock Waves

Chapter 1

Oscillations

1.1 Free Oscillations

1.1.1 Simple Harmonic Motion (SHM)

Oscillations: Oscillation is a repeating motion that occurs when a periodic force acts on the system. Oscillations are periodic motions.

Free Oscillations If the oscillations occur without the action of an external periodic force then such oscillations are called ***free oscillations***.

Simple Harmonic Motion The motion of an object is said to be simple harmonic motion if the restoring force (or acceleration) is directly proportional to the displacement and acts in the direction opposite to that of motion. Motion of the bob of an oscillating pendulum, spring mass system are the best examples of SHM.
In general the equation for the displacement in SHM is given by

$$y = Asin(\omega_0 t + \phi) \tag{1.1}$$

Here ϕ is the initial phase and A is the amplitude of SHM.

1.1.2 Mechanical Simple Harmonic Oscillator and Expression for SHM (Equation for Free Oscillations)

Consider a mass attached to spring of negligible mass which is suspended from a rigid support as shown in the figure. The oscillations in the mass are due to the restoring force developed in the spring.

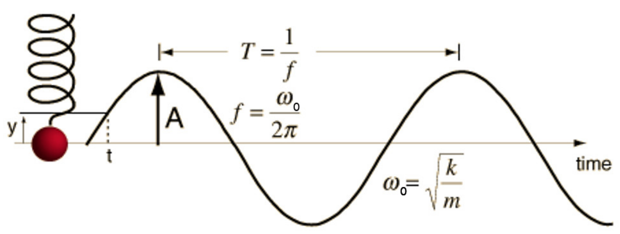

Figure 1.1: Oscillating Spring-Mass system

According to Hooke's Law the restoring force is directly proportional to displacement.

$$f = -ky \tag{1.2}$$

Here k is called spring constant or stiffness factor or force constant. Note : y is used since the displacement is along vertical direction.

$$ma = -ky$$

Applying equation of kinematics of motion the above equation could be written as

$$m\frac{d^2y}{dt^2} = -ky$$

$$\frac{d^2y}{dt^2} = \frac{-k}{m}y$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

$$\frac{d^2y}{dt^2} + \omega_0^2 y = 0 \tag{1.3}$$

Equation 1.3 is the second order homogeneous differential equation for SHM. Here $\omega_0 = \sqrt{\frac{k}{m}}$ is the angular frequency of the oscillations. The solution of equation 1.3 is given by

$$y = Asin(\omega_0 t + \phi) \tag{1.4}$$

Equation 1.4 also represents the equation of motion for free oscillations. A is the amplitude of the SHM and ϕ is the initial phase.

1.1.3 Characteristics of SHM

- Too and Fro motion
- Periodic Motion
- Acceleration or Force is proportional to the displacement.
- Acceleration is in the opposite direction of displacement.
- Restoring force is essential for SHM.

Amplitude (A): The Maximum displacement of the mass from equilibrium position. It can take values $+A$ and $-A$.

Phase Angle and Initial Phase : The value $(\omega t + \phi)$ represents the state of the system and is called phase angle. The angle ϕ is called initial phase.

Angular velocity or Frequency (ω_0): It is the rate of change of angular displacement and is given by $\omega_0 = \sqrt{\frac{k}{m}}$

Frequency (f): Frequency of oscillations is defined as the no. of oscillations per second and is given by $f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

Time Period (T) : It is the time taken to complete one oscillation and is given by $T = \frac{1}{f}$.

Velocity (v): The rate of change of displacement of the oscillating particle is its velocity. Hence we get

$$v = \frac{dy}{dt} = A\omega_0 Cos(\omega_0 t + \phi) \qquad (1.5)$$

The velocity varies from $+A\omega_0$ to $-A\omega_0$.

Acceleration (a): The rate of change of velocity of oscillating particle is its acceleration.

$$a = \frac{dv}{dt} = -A\omega_0^2 Sin(\omega_0 t + \phi) = -\omega_0^2 y \qquad (1.6)$$

Acceleration varies from $+A\omega_0^2$ to $-A\omega_0^2$.

1.2 Springs

1.2.1 Force constant and its Physical Significance

"It is the amount of force exerted when a spring is elongated/compressed by unit length." It determines the stiffness of the string. The SI unit of spring constant unit is Nm^{-1}). The physical significance of k is, if k large then higher force is required for unit extension and if k is small relatively lower force is required for unit extension in the spring.

$$f = kx \qquad (1.7)$$

f is the restoring force, x is the displacement and k is spring constant or stiffness factor.

1.2.2 Natural Frequency

When a body exhibits free oscillations the frequency with which the oscillations occur is called *Natural Frequency*.

1.2.3 Springs in Series and Parallel

Consider two springs of negligible masses and with force constants k_1 and k_2. Let us calculate the effective spring constant when the two springs are connected in series to a mass and when connected in parallel, as shown in the figure 1.2.

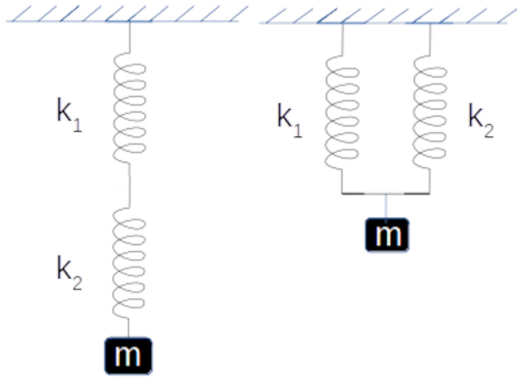

Figure 1.2: Springs in series and parallel

Springs in Series

When springs are in connected in series to a mass the net extension y is given by

$$y = y_1 + y_2 \qquad (1.8)$$

Here y is the total extension, y_1 is the extension in the spring with force constant k_1 and y_2 is the extension in the spring with force constant k_2. If k_s is the effective force constant of the combination and if F is the force applied on the combination then we get

$$\frac{F}{k_s} = \frac{F}{k_1} + \frac{F}{k_2}$$

Thus

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \qquad (1.9)$$

Thus from equation 1.9 effective force constant of the series combination is given by

$$k_s = \frac{k_1 k_2}{k_1 + k_2} \qquad (1.10)$$

Springs in Parallel

When springs are in connected in parallel to a mass the net force F on the system of springs is given by

$$F = F_1 + F_2 \qquad (1.11)$$

Here F_1 is the force on the spring with force constant k_1 and F_2 is the force on the spring with force constant k_2. If

k_p is the effective force constant of the combination and if y is the total extension in the combination then we get

$$k_p y = k_1 y + k_2 y$$

Thus the effective force constant of the parallel combination is given by

$$k_p = k_1 + k_2 \tag{1.12}$$

1.2.4 Applications of Springs

Compression Springs

Introduction : A compression spring is an elastic coil, made of spring steel - its spring characteristic is that it absorbs force or provides resistance.

Structure and Working :

- Compression springs are very effective at building up energy. The compression spring has gaps between its coils in an unloaded state.
- The distance between the coils is reduced when the spring is loaded and compressed.

Uses : A compression spring can be used as a pure energy accumulator, shock absorber, suspensions, vibration, damper or force generator.

Life : To get the longest possible service life from a compression spring, it is important not to overload it.

Leaf Springs

Structure and Working :

- The Leaf Spring is made of arc-shaped slender pieces of steel that are stacked in smaller sizes and bolted together.
- Its construction creates a reinforced bow-like item. It is then attached to the rear axle and the chassis.

The overall purpose of a leaf spring is to provide support for a vehicle. It also creates a smoother ride, absorbing any bumps or potholes in the road.

Uses:

- The leaf springs are used control the height at which the vehicle rides and keep the tires aligned on the road.
- Leaf springs are most useful for train, trucks and vans hauling heavy loads.

1.3 Damped Oscillations

Definition If the amplitude of oscillations decreases with respect to time then they are called damped oscillations. The decay in amplitude of the oscillations is due to the involvement of resistive forces like friction resulting in energy loss. The following are the examples for damped oscillations.

1.3.1 Examples of Damped Oscillations

Spring Mass System With Mass Immersed in a Viscous Liquid

Consider a spring mass system in which the oscillating mass is immersed in a viscous fluid. During the oscillations the viscous force acting on the mass reduces the amplitude progressively.

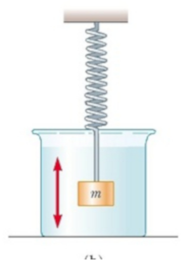

Figure 1.3: Spring Mass - Damped system

1.3.2 Expression for the Decay of the Amplitude in Damped Oscillations

Let us consider a simple harmonic oscillator system damped by viscous damping forces. The damping force is proportional to the velocity of the system. Thus the damping force is given by

$$f_d = -b\frac{dy}{dt} \tag{1.13}$$

here b is a constant that depends on the medium and the shape of the body called Damping Constant. Thus the equation for the damped simple harmonic oscillator could be obtained by adding the damping force term to the Hooke's law.

$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt}$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

The above equation could be written as

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = 0 \tag{1.14}$$

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = 0 \tag{1.15}$$

Here $\gamma = \frac{b}{m}$ called damping ratio and $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency of the system. Thus the general solution for equation 1.14 could be written as

$$y(t) = Me^{\left(\frac{-b}{2m}+\frac{1}{2m}\sqrt{b^2-4mk}\right)t} + Ne^{\left(\frac{-b}{2m}-\frac{1}{2m}\sqrt{b^2-4mk}\right)t} \tag{1.16}$$

For small damping, the above equation could be reduced to

$$y(t) = Ae^{\frac{-b}{2m}t}cos(\omega t + \phi) \tag{1.17}$$

here the frequency of damped oscillations $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$.
The amplitude of damped oscillations is given by

$$Ae^{\frac{-b}{2m}t} \tag{1.18}$$

The amplitude of damped oscillations decreases with increase in time. The frequency of the damped oscillations ω is less than the natural frequency ω_0.

The damping is classified into three types as shown below

Under damping : Oscillations are said to be under damped or weakly damped if the retarding force is weaker than the restoring force. The amplitude of oscillations decreases with respect to time. The condition for damped oscillations is $b^2 < 4mk$.

Over Damping : Oscillations are said to be over damped or heavy damped when the system attains equilibrium state quite slowly without making oscillations. The condition for over damping is given by $b^2 > 4mk$

Critical Damping : When the system approaches equilibrium state quite quickly without making any oscillations is called Critical Damping. The condition for critical damping is given by $b^2 = 4mk$.

The above conditions are as shown in the below graph.

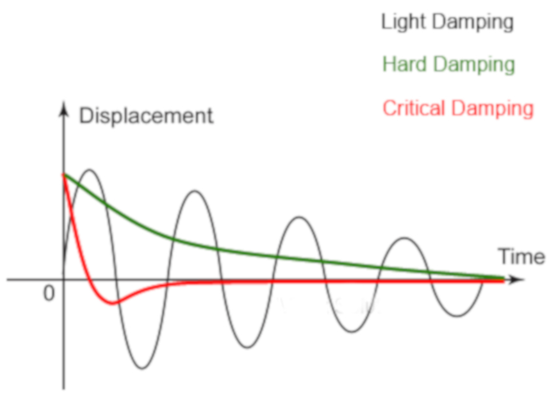

Figure 1.4: Under, Over, Critical damping

1.3.3 Quality factor and its Significance

Definition

The energy loss rate of a weakly damped oscillator is conveniently characterized in terms of a parameter Q called Quality factor. The quality factor is defined as 2π times the ratio of energy stored in the oscillator to the energy lost per time period. Q is also given by

$$Q = \frac{1}{\gamma}\sqrt{\frac{k}{m}} \tag{1.19}$$

This is because $\tau = \frac{1}{\gamma}$ and $\omega = \sqrt{\frac{k}{m}}$.

Significance

If the oscillator is weakly damped then the energy loss per period is relatively small and the Quality factor is much large than unity. Quality factor could be considered as the number of oscillations that the oscillator typically completes before its amplitude decays to a negligible value.

1.3.4 Engineering Applications of Damped Oscillations

Automatic Door Closures

Principle : The Heavily Damped system returns to the equilibrium position very slowly, without any oscillation. Heavy damping occurs when the resistive forces exceed those of critical damping.

Structure and Working :

- One end of the hydraulic damper is attached to the door, and the other end to the door frame.
- When door is opened, the hydraulic door closer pulls the door and politely closes it rather than slamming the door.

- This happens because the closer has a sealed tube which contains a strong spring as a damper.
- It also includes a fluid-filled chamber which releases the pressure to close the door in a slow manner rather than banging it.

Automobile Suspension System

principle The Automobile Suspension System works on the principle of Critical Damping. The damper in the suspension returns to the equilibrium quickly.

Structure and Working

- The automobile suspension consists of a compression spring of suitable damping connected between drive shaft and chassis.
- Spring must be flexible so that if can absorb road shocks.
- But if it is too flexible it may rebound excessively and repeatedly resulting in a a rough ride.
- For effective cushioning even a cylinder-piston damper is also fixed in vehicles.

1.4 Forced Oscillations

1.4.1 Definition and Examples

Forced Oscillations : The oscillations occur that under the action of an external periodic force are called forced oscillations. During forced oscillations the system oscillates with the frequency of the external periodic force.

Examples : Sonometer wire exited by an electromagnet or tuning fork and Resonance air column are the examples of forced oscillations.

1.4.2 Expression for Amplitude and Phase in Forced Oscillations

The forces acting on the system during forced oscillations

1. Restoring force acting in the direction opposite to the displacement.
2. Damping force due to the viscous medium.
3. External periodic force acting on the system.

Thus the equation oscillations in differential form could be written as

$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt} + F_0cos(\omega t)$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0cos(\omega t)$$

The above equation could be written as

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F_0}{m}cos(\omega t) \tag{1.20}$$

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = \frac{F_0}{m}cos(\omega t) \tag{1.21}$$

Here $\gamma = \frac{b}{m}$ called damping coefficient, $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency, $\frac{F_0}{m}cos(\omega t)$ is the applied periodic force and ω is the frequency of the applied periodic force.The above equation could be represented in the complex exponential form as

$$\frac{d^2z}{dt^2} + \gamma\frac{dz}{dt} + \omega_0^2 z = \frac{F_0}{m}e^{i\omega t} \tag{1.22}$$

let us assume the following solution fo

r the above equation

$$z = Ae^{i(\omega t-\phi)} \tag{1.23}$$

Substituting equation 1.23 in the equation 1.22 and simplifying we can obtain the relation for the amplitude and phase as follows.

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2-\omega^2)^2 + (\gamma\omega)^2}} \tag{1.24}$$

$$\tan\phi = \frac{\gamma\omega}{\omega_0^2-\omega^2} \tag{1.25}$$

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1.4.3 Different conditions of forced oscillations

1. If $\omega << \omega_0$: If the frequency of the applied force is very less than the natural frequency Then the equation 1.24 becomes
$$A = \frac{F_0}{m\omega_0^2} \tag{1.26}$$
Hence the system oscillates with frequency ω and its amplitude depends on $\frac{F_0}{m}$ and independent of ω. Equation 1.25 becomes
$$tan(\phi) = \frac{\gamma\omega}{\omega_0^2} \approx 0 \tag{1.27}$$
2. If $\omega = \omega_0$ the equation 1.24 reduces to
$$A = \frac{F_0}{m\gamma\omega_0} = \frac{F_0}{b\omega_0} \tag{1.28}$$
Here the amplitude of the oscillations is maximum since there is no square for ω in the denominator when compared to the previous case. This condition is called resonance. The equation for phase difference could be obtained from equation 1.25 by substituting $\omega = \omega_0$. Since the denominator is zero $tan\phi = \infty$ and hence the phase angle between displacement and the applied periodic force is $\frac{\pi}{2}$.

3. If $\omega >> \omega_0$ This case is significant only when the damping forces are very small (for small γ). The equation 1.24 reduces to

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\gamma\omega)^2 + \omega^4}}$$

$$A = \frac{\frac{F_0}{m}}{\omega^2} = \frac{F_0}{m\omega^2} \tag{1.29}$$

the equation 1.25 becomes

$$tan\phi = -\frac{\gamma}{\omega} \tag{1.30}$$

for small γ the above equation is zero. Thus $tan(\phi) = 0$ and the phase difference between the displacement and the applied periodic force is $-\pi$.

The graph representing the variation of amplitude as a function of driver frequency and damping is as shown in the below figure 1.5.

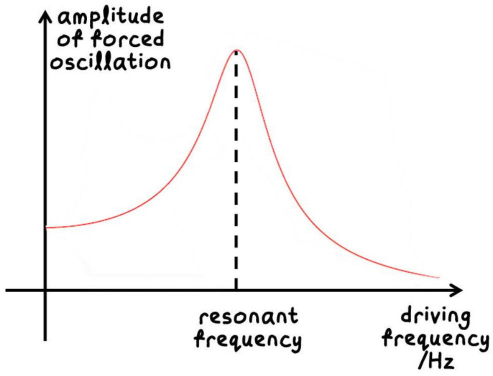


Figure 1.5: Variation of amplitude with driver force frequency

The graph representing the variation of phase with respect to dirver frequency is as shown in the below figure 1.6.

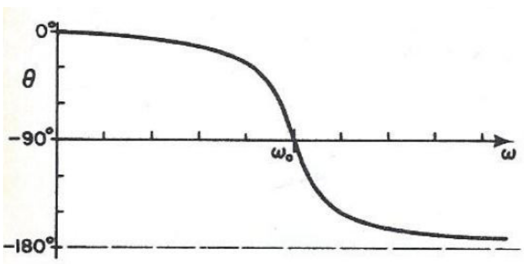


Figure 1.6: Variation of phase with driver force frequecy

1.4.4 Resonance

Consider a system under forced oscillations in which the frequecy of the applied periodic force is varied (tuning). During the course of tuning, if the frequency of the applied periodic force ω matches with the natural frequency ω_0 of oscillations of the system then the amplitude of the oscillations will be maximum and the condition is called resonance.

$$A = \frac{F_0}{m\gamma\omega_0} = \frac{F_0}{b\omega_0} \tag{1.31}$$

As shown in the figure 1.5 the amplitude of oscillations is maximum at resonant frequency.

Examples of Resonance

The following are the examples of resonance in different oscillating systems under forced oscillations.

1. In sonometer when the natural frequency of the stretched string is equal to the frequency of the tuning fork the amplitude of oscillation is maximum.
2. Helmoltz resonator
3. Resonance in LCR circuits, an example for electrical resonance.
4. The absorption of energy by electrons in atoms.
5. Resonance air column.

1.4.5 Sharpness of Resonance

Definition and Significance

During the tuning of oscillating system the rate at which the amplitude varies near resonance depends on damping.

The sharpness of resonance is the rate of change of amplitude with respect to a small change in frequency of the applied external periodic force, at resonance. Mathematically

$$Sharpness of resonance = \frac{\Delta A}{\Delta\omega}$$

Here ΔA is the change in amplitude with respect to $\Delta\omega$ the change in frequency at resonance. The following expression could be derived for sharpness of resonance.

$$Sharpness of resonance = \frac{F_0}{m\gamma\omega_0\omega} = \frac{F_0}{b\omega_0\omega} \tag{1.32}$$

Effect of damping on the sharpness of resonance

The graph representing the variation of amplitude of forced oscillations with respect to damping is as shown in the below figure 1.7.

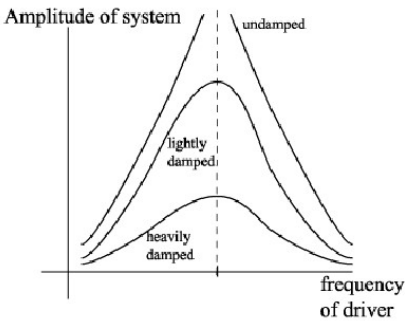


Figure 1.7: Variation of amplitude with damping

With reference to equation 1.32 the maximum amplitude at resonance is a function of damping. Higher the damping lower will be the amplitude at resonance. Thus the sharpness will be higher at lower damping and vise-versa is the significance.

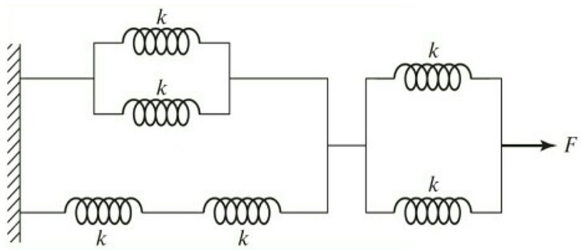
1.5 Model Questions

1. Define SHM and Explain its characteristics.
2. Set up Differential equation for the SHM and arrive at the solution.
3. Derive the equation of motion for Free Oscillations and arrive at the solution.
4. Define force constant and its physical significance.
5. Discuss the effective spring constant of sprigs in series and parallel.
6. Describe damped oscillations with examples.
7. Discuss the theory of damped oscillations and classify damped oscillations in to under damping, critical damping and over damping.
8. Discuss the theory of forced vibrations.
9. Explain the dependence of amplitude and frequency on the applied periodic force in forced oscillations.
10. Describe resonance and its physical significance.
11. Explain the sharpness of resonance.

1.6 Numerical Problems

1. Find the frequency of oscillations of a free particle executing SHM of amplitude 0.35 m if he maximum velocity it can attain is 220 m/s. Ans 100 Hz.
2. Find the displacement at the end of 3 seconds and also the amplitude of oscillations of a body executing simple harmonic motion in a straight line if its period is 10 seconds, and if its velocity is 1 m/s, at the time 2 seconds after crossing the equilibrium. Assume that there are no resistive forces. Ans:Amplitude 5.15 m, Displacement at 3 sec = 4.9 m.
3. Find the frequency of vibration of a sonometer wire which reaches a maximum velocity of 6.28 m/s, when its amplitude of vibration is 1 cm. (Assume free vibrations). Ans: frequency = 100 Hz.
4. Evaluate the resonance frequency of a spring of force constant 2467 N/m, carrying a mass of 100gm. frequency = 25 Hz.
5. Given the force constant as 9.8 N/m for a spring, estimate the number of oscillations it would complete in 1 minute if it is set for oscillations with a load of 89.37 gm. Assume there are no external forces acting on it. Ans : frequency = 100 Cycles / min.
6. A Mass of 10 kg is suspended from the free end of a spring. When set for oscillations, the system executes 100 oscillations in 5 minute. Calculate the force constant of the spring. Ans 2.27 N/m.
7. A mass of 100 kg is mounted on 4 springs each of which has spring constant $4x10^3$ N/m. The motor moves only in vertical direction. Find the natural frequency of the system.Ans : Frequency = 2 Hz.
8. A mass of 4.3 gm is attached to spring of negligible mass of force constant 17 N/m. this mass spring system is executing simple harmonic oscillations. Find out the frequency of the external force which excites resonance in the system. f=10 Hz.
9. An arrangement of identical springs is as shown in the figure 1.8. If the spring constant of each spring is $100Nm^{-1}$ calculate the effective spring constant of the combination. Also calculate the frequency of oscillation of the system when a mass of 1 kg is attached. f=1.677 Hz.

Figure 1.8:



10. A mass of 500 g is attached to a spring and the system is driven by an external periodic force of amplitude 15 N, and frequency of 0.796 Hz. The spring extends by a length of 88 mm under the given load. Calculate the amplitude of oscillations if the resistance co-efficient of the medium is 5.05 kg/s. Ignore the mass of the spring. Ans: Amplitude 0.3 m.

11. Calculate the resonance amplitude of the vibration of the system whose natural frequency is 1000 Hz when it oscillates in the resistive medium for which the value of damping per unit mass is 0.008 rad/s under the action of an external periodic force/unit mass of amplitude 5 N/kg, with tunable frequency. Ans: 0.1 m

12. A 20 gm oscillator with natural angular frequency 10 rad/sec is vibrating in damping medium. the damping force is proportional to the velocity of the vibrator. If the damping coefficient is 0.17, how does the oscillations decay. Ans: Its a case of underdamping.

Chapter 2

Shockwaves

2.1 Mach number

It is defined as the ratio of speed of the object (v_0) to the speed of the sound (v_s) in the surrounding medium. It is a dimensionless quantity. Mathematically Mach number M is given by

$$M = \frac{v_o}{v_s} \tag{2.1}$$

Mach 0.5 represents half the speed of sound and Mach 2 represents twice the speed of sound. The Mach number is named after the scientist Ernst Mach.

2.2 Classification of waves and Mach regimes

The classification of waves is as follows

2.2.1 Types of waves

Acoustics : It is the study which deals with the study of waves through gases, liquids and solids.

Infrasonics : It is the study of waves which have frequencies lower than the audible sound (<20kHz)

Ultrasonics : It is the study of waves which have frequencies higher than the audible sound (>20kHz).

Subsonic : Waves which travel with speeds less than the speed of sound in a medium are called subsonic waves ($M < 1$).

Transonic : The speed over lapping between subsonic and supersonic speeds is the transonic waves ($0.8 < M < 1.2$) .

Supersonic : Waves which travel with speeds greater than the speed of sound in a medium are called supersonic waves ($M > 1$).

2.2.2 Mach regimes

The following is a table representing the regimes of fluid flow of moving object in a fluid with regard to Mach number.

Mach No.	Regime
Subsonic	0.8
Transonic	0.8 - 1.2
Sonic	1.0
Supersonic	1.0 - 5.0
Hypersonic	5.0 - 10.0
High-Hypersonic	>10

2.3 Shock Waves and its applications

2.3.1 Shock Waves

When the speed of a moving object in a medium exceeds the speed of sound in the medium then the wave fronts lag behind the source forming a cone-shaped region with the source at a vertex. The edge of the cone forms a supersonic wave front with an unusually large amplitude called a "shock wave". A sonic boom is heard when shock waves reach an observer. Shock waves generated when a fighter jet attains the speed of sound is as shown in the figure 2.1.

Figure 2.1: Shock Waves

2.3.2 Mach Angle

Mach angle is half of the vertex angle of a Mach cone whose sine is the ratio of the speed of sound to the speed of a moving body.

$$\sin\theta = \frac{v_s}{v_o} = \frac{1}{M} \tag{2.2}$$

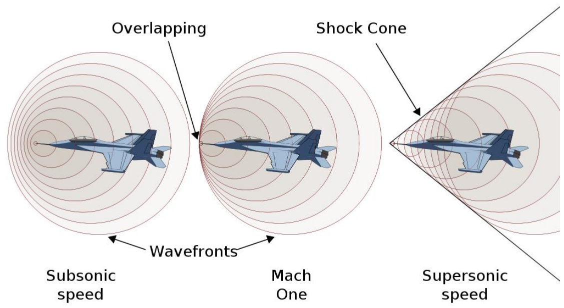


Figure 2.2: Shock Waves schematic

The Mach angle depends on the speed of the object. For transonic speeds Mach angle is 90°and for supersonic speeds Mach angle will be < 90°.

2.3.3 Properties of Shock waves

1. Shock waves travel in a medium with mach (M > 1).
2. Shock waves obey the laws of fluid dynamics.
3. When shock waves pass through a medium the entropy of the system increases.
4. When shock waves pass through a medium the changes are adiabatic.
5. General wave properties cannot be associated with shock waves.

2.3.4 Applications of Shock waves

The following are the applications of the shock waves.

1. In Aerospace the shock waves are used to study the behavior of gases and liquids at high speeds.
2. They are also used in the design of supersonic and hypersonic vehicles
3. They are used in the study of re-entry dynamics and the behavior of spacecraft.
4. Shock waves are used in chemical kinetics to determine and rate of thermal decomposition of NO_2
5. Shock waves are used tn the study of explosions the blast wave signature is a very vital parameter
6. Mach reflection of a shock wave is used to remove micron size dust particles from the surface of silicon wafers.
7. Shock waves are used in medical therapy in orthopedics and for breaking kidney stones.
8. Shock waves are used in pencil industries to impregnate preservatives into wood slats.
9. Shock waves are used in sandal oil extraction.

2.3.5 Shock Tubes

A device used to produce shock waves of required strength in the laboratory is called shock tube. The following are some of the types of shock tubes.

1. Compression Driven
2. Blast Driven
3. Piston Driven
4. Reddy Tube

2.3.6 Reddy Shock Tube

Dr. K.P.J. Reddy and associates developed a miniaturized simple hand-operated piston-driven shock tube named 'Reddy Shock Tube'. The Reddy shock tube is as shown in the figure 2.3.

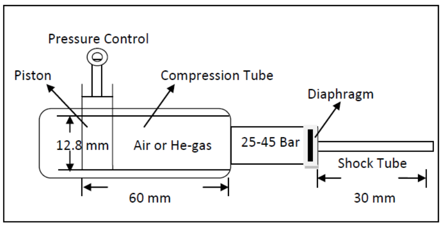


Figure 2.3: Reddy Shock Tube

Construction : Reddy tube operates on the principle of conventional piston driven shock tube is made of simple medical syringe with the following dimensions

- Compression chamber is of length 60mm and dia 12.8mm.
- A plastic piston with plunger with outer dia 12.8mm fitted closely inside the compression chamber.
- O-rings on the piston head prevent the leakage of leaking of gas.
- The driven tube is a 30 mm long SS tube of 1 mm internal diameter and wall thickness of 0.8 mm.
- The compression tube and the shock tube (hypodermic needle) are separated by a 50 μm thick plastic diaphragm.
- The elliptic free end of the hypodermic needle (shock tube) is made circular by grinding and is open to atmospheric air.

Working :

- Diaphragm and the needle are detached from the compression tube.
- the piston is moved outwards to fill the compression chamber with ambient air at atmospheric pressure.
- The diaphragm and the needle are attached to the compression tube.
- The piston is pushed into the compression chamber so that the air inside is compressed and pressure builds.
- As the piston moves the pressure increases and finally the diaphragm ruptures resulting in the very high speed flow of the compressed air.
- Schlieren images of the flow at the exit of the Reddy tube confirm the generation of shock waves.
- Thus shock waves of mach 1.5 to 2.0 are generated at the tip of the needle.

Model Questions

1. Classify the waves in to different types.
2. Define Mach number. Give the classification of waves based on Mach-number.
3. What are Shock Waves. Mention the properties of Shock wave.
4. Define control volume and hence explain the conservation laws as applied to the propagation of shock-waves in a fluid.
5. What is a shock tube? Explain in detail the construction and working of Reddy shock tube and its advantages over other shock tubes.
6. Explain briefly the applications of shock-waves.

Numerical Problem

1. The distance between two pressure sensors in a shock tube is 100mm. Shock waves travel between the two sensors in 100μs. Find the mach number of the shock wave if he velocity of sound under the condition is 330m/s.
2. Calculate the mach angle of the given the speed of source 660m/s and the velocity of the sound in the medium is 330m/s. Ans: 30°

Part II

Module - 2 - Elastic Properties of Materials

Chapter 3

Elastic Properties of Materials

3.1 Introduction

The forces that deform or tend to deform a body are called **deforming forces**. The body which undergoes change in dimensions is called a deformed or strained body. This chapter is intended to study the elastic properties of the materials.

3.2 Elasticity and Plasticity

3.2.1 Elasticity and Plasticity

Elasticity : The property of the body by the virtue of which it tends to regain its shape and size after the deforming forces are withdrawn is called Elasticity and the body is said to be elastic.

Plasticity : The property of the body by the virtue of which it cannot regain its shape and size after the deforming forces are withdrawn is called Plasticity and the body is said to be Plastic.

3.3 Stress and Strain

3.3.1 Stress

The restoring force per unit area is called Stress. It has the unit of pressure (Pa). mathematically

$$Stress = \frac{Restoring\ Force}{Area} = \frac{F}{A}\ Nm^{-2} \tag{3.1}$$

3.3.2 Strain

The fractional change in the dimension of the body is called strain. Mathematically

$$Strain = \frac{Change\ in\ dimensionof\ the\ body}{Original\ dimension\ of\ the\ body} \tag{3.2}$$

3.3.3 Classification of Stress and Strain

- Tensile stress and Linear strain.
- Bulk stress and Volume strain.
- Shearing stress and Shearing strain.

3.4 Stress vs Strain Curve and Hooke's Law

3.4.1 Hooke's Law

For an elastic material the stress is proportional to the strain within the elastic limit. This proportionality is called Hooke's Law. Mathematically

$$Stress \propto Strain$$

$$Stress = Elastic\ Modulus \times Strain$$

Thus the elastic modulus is defined as

$$Elastic\ Modulus = \frac{Stress}{Strain} \tag{3.3}$$

The constant of proportionality is called Elastic Modulus. The unit of Elastic Modulus is same as stress $Nm^{-2} or Pa$. Strain has no unit.

3.4.2 Stress vs Strain curve

Consider a bar or wire of uniform cross section fixed at one end and loaded at the other. The stress-strain relationship could be understood by making a plot of stress vs strain and is called stress-strain diagram. It is as shown in the figure.

1. The region "OA" is the elastic region and Hooke's Law is valid. The point "A" is the "Proportionality Limit".
2. Between the points "A" and "B" the material exhibits elastic behavior. But Hooke's Law is not valid. The point "B" is the elastic limit.
3. Further for any point 'C' the material possess permanent strain and plastic behavior is observed. While unloading the dotted curve is traced.

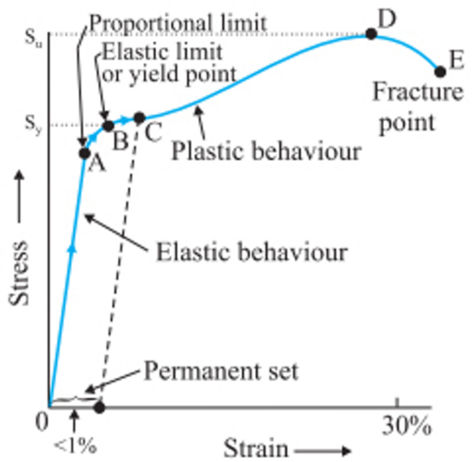


Figure 3.1: Tensile Stress and Linear Strain

4. The point "D" corresponds to the maximum stress that the material can withstand and is called "Point of ultimate strength".

5. At the point "E" material breaks and the body no more exists in single piece. The point "E" is called "Fracture Point". The stress corresponding to the point "E" is called "Breaking Stress".

3.5 Strain Hardening and Softening

3.5.1 Strain Hardening

The strain hardening is the process of making a metal harder by plastic deformation. Consider a material stressed and deformed beyond elastic limit and then unloaded gradually. The curve traced in the graph is "BC" with residual strain "OC".

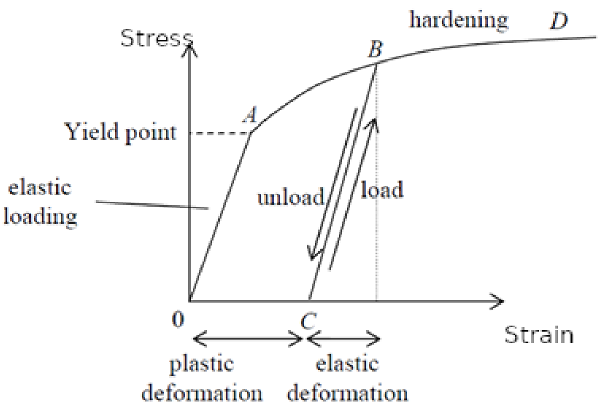


Figure 3.2: Strain Hardening

if the loading is repeated again in steps and the graph is plotted with C as origin curve CB is observed. Thus the material exhibits proportionality to a higher value of stress. Hence the material can withstand higher stresses and is known as strain hardening.

Example : copper-zinc alloy, brass, used for ammunition cartridges and the aluminum-magnesium alloys in beverage cans.

3.5.2 Strain Softening

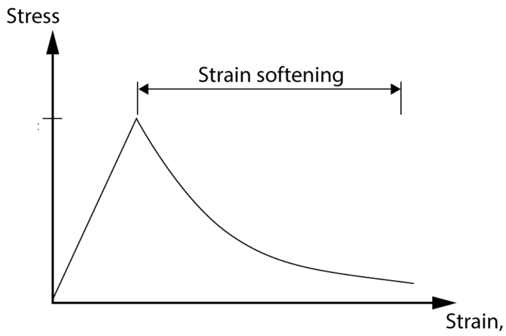


Figure 3.3: Strain Softening

In case of certain materials, when they are stressed beyond elastic region the stress-strain curve takes negative slope. Since the material can withstand only reduced stresses in this region it is called strain softening.

Example : Typically observed at a continuum level in damaged quasi brittle materials, including fibre reinforced composites and concrete.

3.6 Elastic Moduli

3.6.1 Young's Modulus(Y, q or E)

It is defined as the ratio of tensile (Longitudinal) stress to longitudinal (Linear) strain.

$$Y = \frac{Longitudinal\ Stress}{Linear\ Strain}$$

$$Y = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{Ax} \quad pascal \ \ or \ \ Nm^{-2} \tag{3.4}$$

Here F is the force acting on area A, x is the change in length and L is the original length.

3.6.2 Bulk Modulus (K)

It is defied as the ratio of compressive stress to volume strain.

$$k = \frac{Compressive\ Stress}{Volume\ Strain}$$

$$k = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V}\ pascal\ or\ Nm^{-2} \tag{3.5}$$

Here P is the uniform pressure applied, ΔV is the change in volume, V is the initial volume.

3.6.3 Rigidity Modulus (n or η)

It is defied as the ratio of Shearing stress to Shearing strain

$$n = \frac{Longitudinal\ Stress}{Linear\ Strain}$$

$$n = \frac{\frac{F}{A}}{\theta} = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{Ax}\ pascal\ or\ Nm^{-2} \tag{3.6}$$

Here F is the tangential force acting on area A, θ shearing strain or shearing angle, x is the change is the displacement along F and L is the original length.

3.7 Strain Coefficients

3.7.1 Longitudinal Strain Coefficient (α)

It is defined as the longitudinal strain produced per unit stress. The longitudinal strain is given by $\frac{x}{L}$. If T is the applied stress then the longitudinal strain coefficient is given by

$$\alpha = \frac{\frac{x}{l}}{T} = \frac{x}{LT} \tag{3.7}$$

$$\Rightarrow Longitudina\ Strain = x = \alpha LT \tag{3.8}$$

3.7.2 Lateral Strain Co-efficient (β)

If a material of cylindrical shape is subjected to stretching the length increases and the diameter decreases. Hence lateral strain is observed and is defined as the ratio of change in diameter to original diameter.

$$Lateral\ Strain = \frac{d}{D}$$

For an applied stress of T

$$Lateral\ Strain\ Co-efficient = \beta = \frac{d}{TD} \tag{3.9}$$

Thus the later strain in terms of β is given by

$$d = \beta TD \tag{3.10}$$

3.8 Poisson's Ratio (σ)

It is defined as the ratio of Lateral Strain to Longitudinal strain within the elastic limit.

$$\sigma = \frac{\frac{d}{D}}{\frac{x}{L}} = \frac{\frac{d}{TD}}{\frac{x}{TL}} = \frac{\beta}{\alpha} \tag{3.11}$$

3.8.1 Expressions for Y, k and n in-terms of α and β

The relations are as follows.

$$Y = \frac{1}{\alpha} \tag{3.12}$$

$$n = \frac{1}{2(\alpha+\beta)} \tag{3.13}$$

$$K = \frac{1}{3(\alpha-2\beta)} \tag{3.14}$$

3.9 Relation between Elastic Moduli

3.9.1 Relation between Y, n and σ

Consider an elastic material in the shape of cube which is glued to a rigid surface as shown in the figure.

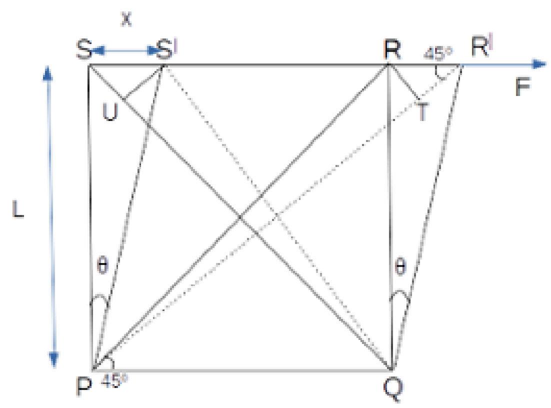


Figure 3.4: Shearing

Let α and β the longitudinal and lateral stress coefficients. The top surface shears through a distance x due to the applied tangential force F on area A. Hence PR increases to PR'. The shearing strain could be considered as due to both longitudinal strain along PR and Lateral strain along QS.

$$TR' = T(PR)\alpha + T(QS)\beta$$

$$TR' = T(PR)(\alpha+\beta)$$

Because $PR = QS$ and also using Pythagoras theorem we get $PR = L\sqrt{2}$

$$TR' = TL\sqrt{2}(\alpha + \beta) \tag{3.15}$$

Consider the angle of shear ϕ to be very small and hence $\angle PRQ = \angle PR'Q = 45°$. Thus from Right angled $\Delta^{le} RR'Q$, we have

$$Cos(45^0) = \frac{TR'}{RR'}$$

$$TR' = \frac{RR'}{\sqrt{2}} = \frac{x}{\sqrt{2}} \tag{3.16}$$

Substituting the above in equation 3.15 we get

$$\frac{x}{\sqrt{2}} = TL\sqrt{2}(\alpha + \beta)$$

$$\frac{TL}{x} = \frac{1}{2(\alpha + \beta)}$$

$$\frac{T}{\frac{x}{L}} = \frac{1}{2\alpha\left(1 + \frac{\beta}{\alpha}\right)} \tag{3.17}$$

We know that $\frac{1}{\alpha} = Y$, $\frac{x}{L} = \theta$, $\frac{\beta}{\alpha} = \sigma and \frac{T}{\theta} = n$. Hence equation 3.17 becomes

$$n = \frac{Y}{2\left(1 + \frac{\beta}{\alpha}\right)}$$

Hence the relation between $Y, n\ and\ \sigma$ is given by

$$Y = 2\ n\ (1 + \sigma) \tag{3.18}$$

3.9.2 Relation between Y, K and σ

The relation between Y, K and σ is as follows.

$$Y = 3K(1 - 2\sigma) \tag{3.19}$$

3.9.3 Relation between Y, K and n

$$Y = \frac{9Kn}{3K + n}$$

The relation between Y, K and n is as follows. Here Y is Young's Modulus, K is Bulk Modulus and n is Rigidity Modulus.

3.9.4 Limiting vaues of σ

Consider the relations

$$Y = 2n(1 + \sigma) \tag{3.20}$$

$$Y = 3K(1 - 2\sigma) \tag{3.21}$$

equating equations 3.20 and 3.21 we get

$$2n(1 + \sigma) = 3K(1 - 2\sigma) \tag{3.22}$$

- If $\sigma > 0.5$ then LHS will be "+Ve" and RHS will be "-Ve".
- If $\sigma < -1$ then LHS will be "-Ve" and RHS will be "+Ve".
- Since both sides represent Young's Modulus they must result in the same positive value.
- Hence for both sides to be "+Ve" the σ can take values in the range $-1 < \sigma < 0.5$.
- Since σ cannot take negative values $0 < \sigma < 0.5$.
- Thus the value of σ rages from 0 to 0.5.

3.10 Bending of Beams

A beam is a bar or rod of uniform cross section whose length is much greater as compared to its other dimensions.These are used in buildings to support roofs and in bridges to support the load of vehicles passing over them.

3.10.1 Assumptions

1. The weight of the beam is negligible compared to the load applied.
2. There are no shearing forces.
3. The cross section and thus geometrical MI of the beam remains unaltered.
4. The curvature of the beam is small.

3.11 Neutral surface and Neutral axis

Whenever a beam is subjected to load it is bent longitudinally. The longitudinal filaments on the convex side of the beam are elongated and those on concave side are compressed. In between these filaments there is a filament which is neither elongated nor compressed and is called Neutral filament. The length of the neutral filaments remains unchanged even after loading the beam. ***Neutral surface*** *is that layer of a uniform beam which does not undergo any changes in its dimensions when the beam is subjected to bending within the elastic limit.* ***Neutral axis*** *is a longitudinal line of intersection of neutral surface and plane of bending*.

3.12 Types of Beams

Beams are classified into the following five types

1. A Simply supported beam is a bar resting upon supports at its ends and this is the most commonly used.

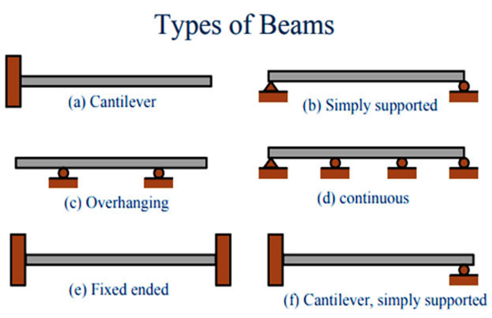


Figure 3.5: Types of Beams

2. A Continuous beam is a bar resting upon more than two supports
3. A Cantilever beam is a beam whose one end is fixed and other end is free.
4. A Fixed ended beam is beam whose both ends are fixed.
5. A Overhanging beam has its one or both ends stretching out past its supports.

3.13 Bending Moment and Expression

3.13.1 Bending Moment

Consider a cantilever beam of uniform cross-section which is bent by applying load. The moment of applied couple due to which the beam bends longitudinally is called **Bending Moment**.

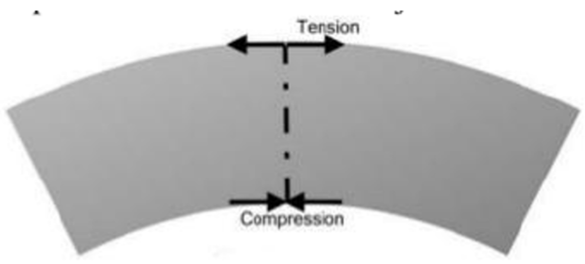


Figure 3.6: Bending of Beam

Definitions

The opposing moment developed in the material due to elasticity is called **Restoring Moment**.

3.13.2 Expression for Bending Moment

Introduction : Consider a beam of uniform cross-section. Consider three layers AB, CD and EF be three layers. Before applying the load $AB = CD = EF$. The beam bent by applying load and is as shown in the figure 3.7. On bending AB extends to $A^{'}B^{'}$ and EF contracts to $E^{'}F^{'}$. The layer CD is the neutral surface. The length CD forms an arc of a circle of radius R and subtends and angle θ at the center. The layer $A^{'}B^{'}$ is concentric to CD with radius $R + r$.

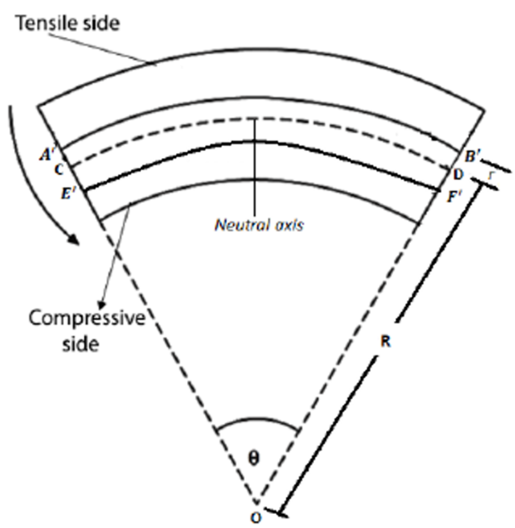


Figure 3.7: Sectional View of the Beam

From the figure $CD = R\theta$ and $A'B' = (R + r)\theta$. The change in length of the layer AB is given by

$$A'B' - AB = A'B' - CD = R\theta - (R + r)\theta$$

Thus the change in length is give by

$$Change\ in\ Length = R\theta - R\theta + r\theta = r\theta$$

The original length $AB = CD = R\theta$, Thus the linear Strain is given by

$$Liner\ Starin = \frac{r\theta}{R\theta} = \frac{r}{R} \tag{3.23}$$

Let Y be the Young's Modulus of the material. According to Hooke's Law,

$$Longitudinal\ Stress = Y \times Linear\ Strain \tag{3.24}$$

the longitudinal strain is also given by $\frac{F}{a}$. Thus 3.24 could be written as

$$\frac{F}{a} = Y \times Linear\ Strain$$

$$F = \frac{Yar}{R} \tag{3.25}$$

Moment of Force of a layer with respect to neutral layer is given by

$$Moment\ of\ Force = F \times \perp Distance$$

$$Moment\ of\ Force = F\ r = \frac{Yar^2}{R} \tag{3.26}$$

The Bending moment is the moment of force acting on the entire beam and is given by

$$Bending\ Moment = \sum \frac{Yar^2}{R} = \frac{Y}{R}\sum ar^2 \tag{3.27}$$

Here $\sum ar^2 = I_g$ is called Geometrical Moment of Inertia of inertia.

$$Bending\ Moment = \frac{Y}{R} I_g \tag{3.28}$$

Thus the expression for Bending Moment.

3.13.3 Geometrical MI of Rectangular and Circular Beams

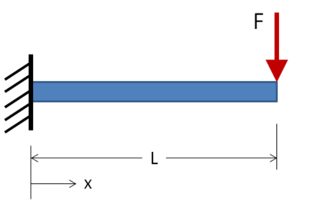
The Geometrical moment of inertia I_g depends of the geometrical shape of beam's cross-section.

1. For a beam of rectangular cross-section of breadth b and thickness d, $I_g = \frac{bd^3}{12}$.
2. For a beam of circular cross-section of radius x, $I_g = \frac{\pi x^4}{4}$.

3.14 Cantilever and I Section Girders

3.14.1 Cantilever

Definition: The cantilever is a beam fixed at one end and free at the opposite end.

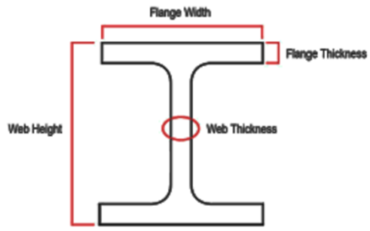


Structure and Purpose : It distributes the load back to support that force against a flash and shear stress. Cantilever construction allows overhanging structures without additional supports and bracing.

Applications : Typically it is firmly attached to a flat vertical surface like wall and extends from it. outwards. A cantilever beam allows the creation of a bay window, some bridges, balconies, truss, or slab.

3.14.2 I Section Girders

Definition I-shaped girders are made like the I-beam section and are composed of couple of load bearing flanges and web.



Structure and Purpose The top and bottom flanges of the "I" shape provide horizontal support and resistance against bending, while the vertical web connects the two flanges and provides resistance against shearing forces. The "I" shape of steel girders and rails provides structural efficiency, material efficiency, and cost-effectiveness.

Uses They are used in building of pulls, fly overs etc.They are also used in construction of public shades and public places.

3.15 Types of Elastic Materials

- Linear Elastic Materials : On loading exhibits a linear graph between of stress vs strain passing through the origin. Eg. Metals.
- Non-Linear Elastic Material : On loading Exhibits non linear graph of stress vs strain passing through the origin. Eg. Large strain rubber or Plastic.
- Cauchy-elastic material : In this material the stress at each point is determined only by the current state of deformation with respect to an arbitrary reference configuration.
- Hyper-elastic materials : They are conservative models that are derived from a strain energy density function.
- Hypo-elastic Materials : hypo-elasticity implies that stress is not derivable from an energy potential.
- Elastomers : Rubbery material composed of long chain-like molecules (polymers) that are capable of

recovering their original shape after being stretched to great extents. eg. Polyurethane.

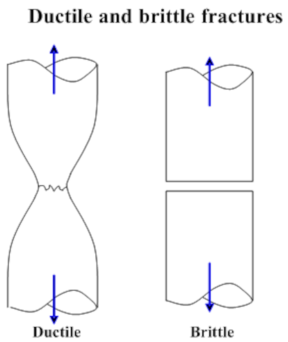
- Some Examples of Elastic materials are Thermo-Plastic Elastomer, Silicone Rubber
- Elastin, Poly Amide (Nylon), Natural Gum, Wool, Lycra.

3.16 Elastic Fatigue and Fracture

Consider an Elastic material which is subjected to number of cycles of loading and unloading. Thus the material experiences repeated strain and relief. During the course the material experiences " Elastic tiredness" or "Elastic Fatigue". A material experiencing fatigue develops cracks which grow over cycles and the materials undergoes fracture and hence called *fatigue failure*. Thus the material(part) fails during the operation. Hence fatigue is an important concern in the design of some mechanical instruments and devices like Engine Pistons, Suspension springs etc.

Fracture is classified into two types.

- Brittle fracture
- Ductile fracture



3.16.1 Brittle Fracture

Brittle fracture means fracture of material without or with very small plastic deformation before fracture. Eg. Rock, concrete, and cast iron. Such materials are called brittle materials.

3.16.2 Ductile Fracture

Ductile fracture is a type of failure seen in readily deformable (malleable) materials and is characterized by extensive plastic deformation or necking that occurs before the material finally cracks or breaks apart. Eg. Aluminium , Copper. Such materials are called ductile materials.

3.17 Stress Concentration and Concentration Factor

3.17.1 Stress Concentration

A stress concentration is a point in a part where the stress is significantly greater than its surrounding area. Stress concentrations occur as a result of irregularities in the geometry or within the material of a component structure that cause an interruption of the stress flow. These interruptions typically arise from discontinuities such as holes, grooves, notches and fillets. Stress concentrations may also be caused by accidental damage such as nicks and scratches.

3.17.2 Concentration Factor

A stress concentration factor (K_t) is a dimensionless factor that is used to quantify how concentrated the stress is in a mechanical part. It is defined as the ratio of the highest stress in the part compared to a reference stress (Nominal Stress).

$$K_t = \frac{S_{max}}{S_{ref}} \tag{3.29}$$

The following are the characteristics of stress factor.

3.18 Factors Affecting Fatigue

3.18.1 Surface Effect

There are always uneven machining marks on the machinated surface. These marks are equivalent to tiny gaps, which cause stress concentration on the surface of the material, thus reducing the fatigue strength of the material.

3.18.2 Design Effect

Any notch or geometrical discontinuity can act as a stress raiser and fatigue crack initiation site; these design features include grooves, holes, keyways, threads, and so on.The sharper the discontinuity (i.e., the smaller the radius of curvature),the more severe the stress concentration.

3.18.3 Environmental Effects

Two types of environment-assisted fatigue are thermal fatigue and corrosion fatigue.

Thermal Fatigue

Thermal fatigue is normally induced at elevated temperatures by fluctuating thermal stresses.

Corrosion Fatigue

Failure that occurs by the simultaneous action of a cyclic stress and chemical attack is termed corrosion fatigue. Corrosive environments have a deleterious influence and produce shorter fatigue lives.

3.19 Model Questions

1. State Hooke's Law and Explain Stress Vs Strain curve with the help of a neat sketch.
2. Discuss Strain Hardening and Softening.
3. Define Elastic Modulus and Explain the three types of Elastic Moduli.
4. Explain Longitudinal and Lateral Strain coefficients and hence define Poisson's ration.
5. Derive the relation between y, n and σ.
6. Discuss the limiting values of Poisson's Ratio.
7. Define Beam and explain the classification of beams.
8. Define Bending Moment, Restoring Moment, Neutral Surface, Neutral Axis.
9. Derive an expression for the Bending Moment of a beam.
10. Describe Cantilever and I section Girder with the held of neat sketches.
11. Discuss the types of elastic material.
12. Elucidate the stress concentration and concentration factor.
13. Discuss the Surface, Design and Environment factors affecting Fatigue.

3.20 Numerical Problems

1. Consider a steel wire of radius 0.13 mm and length 2m. If the wire is rigidly fixed at one and loaded at the other with a mass of 1.5 kg the extension observed is 2 mm. Calculate the Young's Modulus of the material of the wire. Ans : 200 GPa
2. Calculate the force required to produce an extension of 1 mm in steel wire of length 1 m and diameter 1 mm. Given Y = 100 GPa. Ans: F = 157.08 N.
3. A solid lead sphere of radius 10.3 meter is subjected to a normal pressure of $10 Nm^2$, acting all over the surface. Determine the change in its volume. Ans:$106 Nm^2$.
4. A bar subjected to a tensile load 55 KN. Bar diameter=31 mm; Gauge length=300mm; extension=.115mm; change in diameter= 0.00367mm. Find: Poisson's ratio, Young's modulus, Bulk modulus, and modulus of rigidity. Ans: Poisson's Ratio σ = 0.308, Y = 190 GPa, K = 165 GPa, η = 72.7 GPa.
5. 5. A block of gelatin is 60 mm by 60 mm by 20 mm when unstressed. A force of 0.245 N is applied tangentially to the upper surface causing a 5 mm displacement relative to the lower surface. The block is placed such that 60X60 comes on the lower and upper surface. Find the shearing stress, shearing strain and shear modulus. Ans: Shearing Stress = 68.1 Nm^2, Shearing strain = .25, Shear Modulus or Rigidity Modulus = 274.4 Nm^2.

Part III

Module - 5 - Material Characterization and Instrumentation Technique